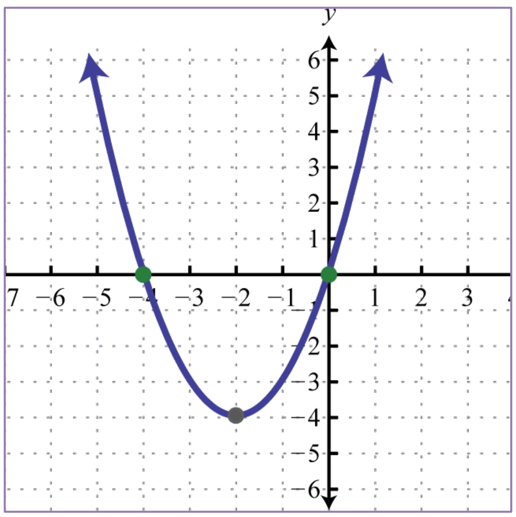
Formal vs. Informal Reasoning

Solving the quadratic equation: $y = ax^2 + bx + c$

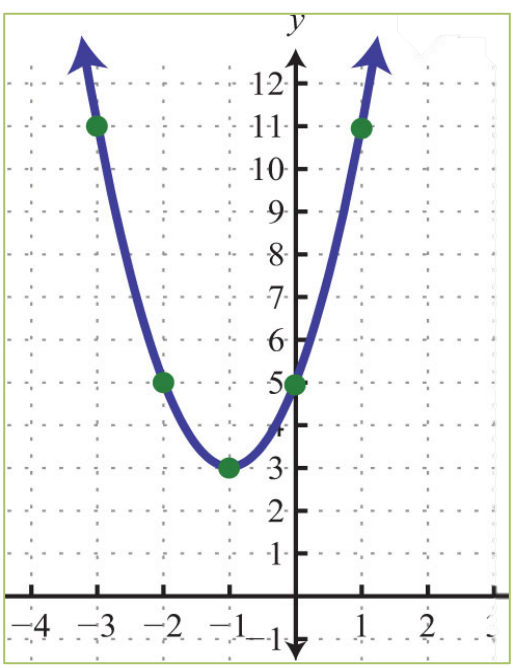
Informal Reasoning



Graphing an equation on the *xy*-plane can help find the roots of a quadratic equation, i.e. where the graph passes through the *x*-axis.

On the graph to the left, it is possible to identify where those roots exist by simply looking at the graph.

Formal Reasoning



In some cases, visually representing an equation to solve for the roots will not be adequate. In these cases, mathematic manipulation will need to be applied to solve for the roots.

Start with the given equation:
$y = ax^2 + bx + c$

The end goal is to find the *x*-value when y = 0. We will set the equation to equal zero and follow the formal steps below.

1. Move the constant to the right side of the equation

$$ax^2 + bx + c = 0$$
$$-c \quad -c$$
$$ax^2 + bx = -c$$

2. Divide by a so x^2 has a coefficient of 1

$$\frac{ax^2 + bx}{a} = \frac{-c}{a}$$

$$x^2 + \frac{bx}{a} = \frac{-c}{a}$$

3. Complete the square

$$\left(\frac{1}{2} * \frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2}$$

$$x^2 + \frac{bx}{a} + \frac{b^2}{4a^2} = \frac{-c}{a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} + \frac{b^2}{4a^2}$$

4. Multiply $\frac{-c}{a}$ by $1 = \frac{4a}{4a}$ to make a common denominator.

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-c}{a} * \frac{4a}{4a} + \frac{b^2}{4a^2}$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac}{4a^2} + \frac{b^2}{4a^2}$$

5. Combine right side terms

$$\left(x + \frac{b}{2a}\right)^2 = \frac{-4ac + b^2}{4a^2}$$

6. Take the square root of both sides to cancel the left side squared term

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{-4ac + b^2}{4a^2}}$$

7. Simplify the radical

$$x + \frac{b}{2a} = \pm\frac{\sqrt{-4ac + b^2}}{\sqrt{2a * 2a}}$$

$$x + \frac{b}{2a} = \pm\frac{\sqrt{-4ac + b^2}}{2a}$$

8. Isolate x; subtract $\frac{b}{2a}$ from side $-$

$$-\frac{b}{2a} \qquad -\frac{b}{2a}$$

$$x = \pm\frac{\sqrt{-4ac + b^2}}{2a} - \frac{b}{2a}$$

9. Simplify the right side

$$x = \frac{\pm\sqrt{-4ac + b^2} - b}{2a}$$

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

Using the above formal reasoning, we can conclude that the equation

$$\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$$

will produce the roots for the quadratic equation $y = ax^2 + bx + c$.