Making Connections: Distance, Circles, & the Pythagorean Theorem

Equations

Distance: $d = \sqrt{(x2 - x1)^2 + (y2 - y1)^2}$

Standard Equation of a Circle: $(x-a)^2 + (y-b)^2 = r^2$ where (a,b) is center

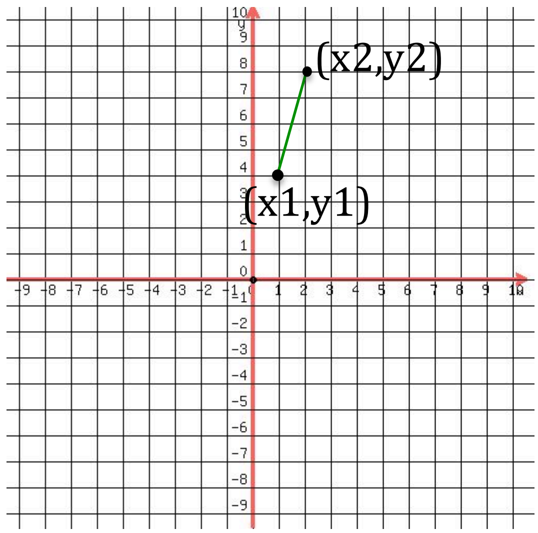
Pythagorean Theorem:$A^2 + B^2 = C^2$

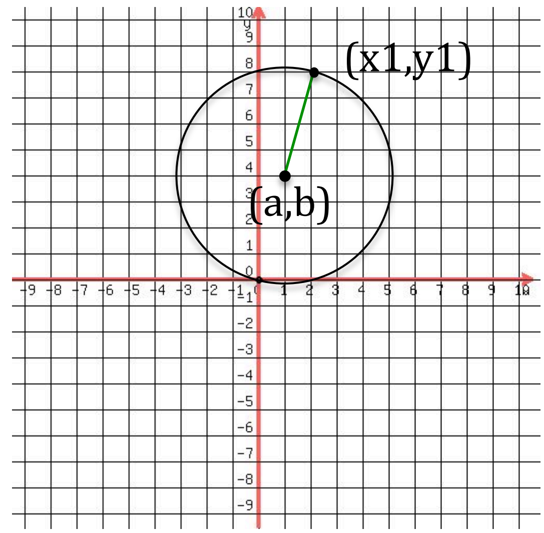
Connecting Formulas

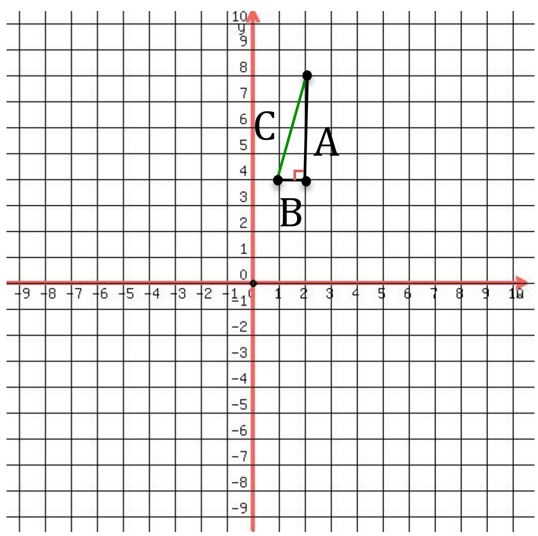
The **Distance** equation can be manipulated to parallel the **Standard Equation of a Circle** formula. The distance from one point: (x1,y1) to another point: (x2, y2) can be found by the equation: d = $\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$. If we look at a circle and name the center point: (x1, y1) and outer point on the circle: (x2, y2), we find that the equation will find the *distance* from the center to the outer edge. This *distance* is also known as the radius of the circle. We can rename the distance variable, "d", as "r" for radius. We will now have r = $\sqrt{(x2 - x1)^2 + (y2 - y1)^2}$. If we square both sides, our new equation will be: $r^2 = (x2 - x1)^2 + (y2 - y1)^2$. As mentioned above, we named the center as (x1, y1). The standard notation for a circle is to name the center of the circle: (a,b) and outer point of the circle (x,y). If we substitute this into our manipulated distance formula, we find that: $r^2 = (x - a)^2 + (y - b)^2$. This now represents the Standard Equation of a Circle. The radius of a circle is no more than the distance from the center of a circle to the outer edge, giving us the liberty to use either equation interchangeably.

The **Standard Equation of a Circle** can also be manipulated to form the **Pythagorean Theorem**. We know from above that the radius of a circle is the distance from the center: (a,b) to another point on the circle: (x1, y1). If we break down the formula: $r^2 = (x - a)^2 + (y - b)^2$, we find that (x – a) is the distance from one x-value to the other. (See above where the distance formula describes this as (x2-x1).) We will rename this distance as A. We can do the same with the y-values, using the value (y – b) as the distance between the y values and naming it B. (See above where this distance is also named (y2 – y1) in the distance formula). Substituting in these new variables, our equation now reads: $r^2 = A^2 + B^2$. Because we found the distance between both x's (vertical length) and y's (horizontal length), these values will intersect at the right angle. We can now replace the *radius* with C representing the hypotenuse of a triangle.
Our final formula now reads: $A^2 + B^2 = C^2$, where A is distance between x's and B is the distance between y's. This creates the sides of a right triangle and makes C the hypotenuse.

<u>GRAPHS</u>







As we see from the graphs, the green line can be found using any of the three formulas where:

Side A = (x2 - x1) = (x1 – a)
Side B = (y2 – y1) = (y1 – b)
Side C = distance(d) = radius(r)

Each equation finds the length in both the x and y direction. This creates a right triangle as you see in the picture beside. Each side is then squared to produce the square of hypotenuse or C^2 in the equation.

We can further extend this concept the to find the midpoint of a line. Using the formula: $(\frac{x1+x2}{2}, \frac{y1+y2}{2})$ we can find the mid point of a line by taking the average of the sum of the two x-values then doing the same for the y-values. This concept can then be tied to the standard equation of a circle. The diameter of a circle can be input into the midpoint formula to find the center point of the circle.

This concept is helpful when plotting lines on a graph. For example, when given specific end points of an equation to graph, the midpoint can be a guide to adding another reference point on the graph's plot. When drawing a circle, the center is needed to help draw radii and compete the graph's image. Students can also check that this truly is the mid-point by conducting the distance formula with the midpoint and endpoint 1, as well as the midpoint and endpoint 2. They will find that both of these will be equal, proving that the midpoint formula holds true.